Journée d'étude PANORisk

Mean Field Game and Heterogenous & Principal Agent Models

Mardi 28 Mars 2017

Université du Maine – Le Mans
Institut du Risque et l’Assurance du Mans
Salle des conférences

9h30-10h00 : Accueil et Café

10h00-10h50
Alain BENSOUSSAN (University of Texas-Dallas)
Control Problems on the space of square integrable random variables and mean field theory

10h50-11h40
Nizar TOUZI (Ecole Polytechnique-Université Paris-Saclay)
Continuous-time Principal-Agent problem: a stackelberg stochastic differential game.

11h40-12h30
René AID (Université de Paris-Dauphine)
The coordination of centralised and distributed generation.

12h30-14h30 : Déjeuner

14h35-15h25
Huyên PHAM (Université de Paris-Diderot)
Robust Markowitz portfolio selection under ambiguous covariance matrix.

15h30-16h20
Boualem DJEHICHE (KTH Royal Institute of Technology - Stockholm)
A risk based approach to the Principal-Agent problem.

16h30-16h50 : Clôture de la journée - Café

Organisation :
Anis MATOUSSI, directeur de l’Institut du Risque et de l’Assurance
Alexandre BROUSTE, professeur au LMM
**Abstracts**

**René AID** (Université de Paris-Dauphine) : The coordination of centralised and distributed generation

**Abstract**: This paper analyses the interaction between centralised carbon emissive technologies and distributed intermittent non-emissive technologies. In our model, there is a representative consumer who can satisfy her electricity demand by investing in distributed generation (solar panels) and by buying power to a centralised firm at a price he set up. Distributed generation is intermittent and induces an externality cost to the consumer. The firm provides non-random electricity generation subject to carbon tax and to transmission costs. The objective of the consumer is to satisfy her demand while minimising investment costs, payment to the firm and intermittency cost. The objective of the firm is to satisfy consumer's residual demand while minimising investment costs, demand deviation costs and maximising payment from the consumer. Investment decisions are formulated as McKean-Vlasov control problems with stochastic coefficients. We provide explicit, price model-free solutions to the optimal decision problems faced by each player, the solution of the Pareto optimum and the Stackelberg equilibrium where the firm is the leader. We find that, from the social planner point of view, carbon tax or transmission costs are necessary to justify a positive share of distributed capacity in the long-term, whatever the respective investment costs of both technologies are. The Stackelberg equilibrium is far from the Pareto equilibrium, leading to a much larger share of distributed energy and to a much higher price for centralised energy.

**Alain BENSOUSSAN** (University of Texas-Dallas) : Control Problems on the space of square integrable random variables and mean field theory

**Abstract**: We implement the idea of P.L. LIONS to write the master equation of mean field theory as a PDE with a space variable which is in the Hilbert space of square integrable random variables. A major advantage is that the formulation of the PDE is much more condensed with this approach, compared with that of using the Wasserstein space. But we explore the idea from the starting point, and consider directly control problems for dynamic systems, whose state space is the Hilbert space of random variables. The method is extremely efficient for first order equations (no randomness except in the elements of the Hilbert space). In the case of additional Wiener processes, the situation is more delicate because of several sources of randomness. We show that the method works for the local noise. A future work will be to explore the common noise.

**Boualem DJEHICHE** (KTH Royal Institute of Technology - Stockholm) : A risk based approach to the Principal-Agent problem

**Abstract**: We suggest a method of characterizing optimal contracts in the classical setting of Principal-Agent problem under hidden action for time inconsistent payoff functions. We consider two different models; Hidden Action in the weak formulation and Hidden Contract in the strong formulation. In the first model the agent has full information about the mechanisms behind the cash-flow and the principal wishes to minimize his/her mean-variance
payoff. In the latter model the agent does not know the structure of the cash-flow and has to protect herself from high levels of risk by an additional participation constraint of variance type. To illustrate the results we consider a "fully solved" example in the linear quadratic setting.

Huyên PHAM (Université de Paris-Diderot) : Robust Markowitz portfolio selection under ambiguous covariance matrix

Abstract: The Markowitz mean-variance portfolio selection problem, initially considered in a single period model, is the cornerstone of modern portfolio allocation theory. Investment decisions rules are made according to the objective of maximizing the expected return for a given financial risk quantified by the variance of the portfolio, and lead to the concept of efficient frontier, which proposes a simple illustration of the trade-off between return and risk. In this talk, we propose a robust version of the continuous-time Markowitz portfolio selection problem when the model uncertainty carries on the covariance matrix of multiple risky assets.

This problem is formulated into a min-max mean-variance problem over a set of non-dominated probability measures that is solved by a McKean-Vlasov dynamic programming approach, which allows us to characterize the solution in terms of a Bellman-Isaacs equation in the Wasserstein space of probability measures. We provide explicit solutions for the optimal robust portfolio strategies and illustrate our results in the case of uncertain volatilities and ambiguous correlation between two risky assets. We also derive the robust efficient frontier in closed-form. We obtain a lower bound for the Sharpe ratio of any robust efficient portfolio strategy, and compare the performance of Sharpe ratios for a robust investor and for an investor with a misspecified model.

Nizar TOUZI (Ecole Polytechnique-Université Paris-Saclay) : Continuous-time Principal-Agent problem: a stackelberg stochastic differential game

Abstract: We provide a systematic method for solving general Principal-Agent problems with possibly infinite horizon. Our main result reduces such Stackelberg stochastic differential games to a standard stochastic control problem, which may be addressed by the standard tools of control theory. Our proofs rely on the backward stochastic differential equations approach to non-Markovian stochastic control, and more specifically, on the recent extensions to the second order case. The infinite horizon setting requires an extension of second order BSDEs to the random horizon setting.